MECS Poststratification Project
Adjusting Weights using a Census Control Total

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One of the several parts of the MECS Poststratification Project is a ratio adjustment of the MECS sample. The ratio adjustment should improve the estimates of all variables correlated with cost of energy. Each MECS establishment will have a new adjusted weight such that the MECS sample estimate of the cost of energy equals the 2002 Census control total.

The adjustment process, conceptually, is ratio adjusting the weighted portion of the sample to equal the total amount from nonselected cases in the population. In otherwords, certainty cases remain self representing, and weighted cases have their self-representing portion unaltered, but their weighted portion changed. The value of all adjusted weights will be at least one, even where the sample overestimates the control total. Letting w_i be the existing weight, then the adjusted weight is $w_{ADJ,i} = 1 + K(w_i - 1)$, where $K = \frac{\sum_{i=1}^{N} x_i - \sum_{i=1}^{n} x_i}{\sum_{i=1}^{n} (w_i - 1) x_i}$. Where the cell contains only certainties of weight 1, no adjustment is made.

Derivation:

We are trying to equate the estimate of total cost of energy from MECS with it counterpart in Census. Therefore, the sum of the weighted sample values of cost of energy should equal the sum of all unweighted values from the Census.

Initially, we have

$$\sum_{i=1}^{N} x_i \neq \sum_{i=1}^{n} w_i x_i \ . \qquad \textbf{(1)}$$

where:

w_i is the current weight of unit_i (sampling weight may already be adjusted)
x_i is the cost of energy of unit_i , from the census
N is the stratum count from the census
n is the sample count in the strata after poststratification, and $n \leq N$

We would like to adjust the weights such that

$$\sum_{i=1}^{N} x_i = \sum_{i=1}^{n} w_{ADJ,i} x_i \qquad \textbf{(2)}$$

where $w_{ADJ,i}$ is the adjusted weight and $w_{ADJ,i} \geq 1$

We can do this by first by decomposing the weighted sum of the cost of energy into its self representing piece and the non self representing (weighted) piece.

$$\sum_{i=1}^{N} x_i \neq \sum_{i=1}^{n} w_i x_i \qquad \text{original inequality (1)}$$

$$\sum_{i=1}^{N} x_i \neq \sum_{i=1}^{n} w_i x_i + \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} x_i$$

$$\sum_{i=1}^{N} x_i \neq \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} w_i x_i - \sum_{i=1}^{n} x_i$$

$$\sum_{i=1}^{N} x_i \neq \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} (w_i - 1) x_i \qquad \textbf{(3)}$$

Resolve inequality with coefficient *K* to the non self representing piece.

$$\sum_{i=1}^{N} x_i = \sum_{i=1}^{n} x_i + K\sum_{i=1}^{n}(w_i - 1)x_i \qquad \textbf{(4)}$$

$$\sum_{i=1}^{N} x_i = \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} K(w_i - 1)x_i$$

$$\sum_{i=1}^{N} x_i = \sum_{i=1}^{n} [x_i + K(w_i - 1)x_i]$$

$$\sum_{i=1}^{N} x_i = \sum_{i=1}^{n} [1 + K(w_i - 1)]x_i$$

Notice this in in the form of $\sum_{i=1}^{n} w_{ADJ,i}\, x_i$

where the $w_{ADJ,i} = 1 + K(w_i - 1)$ **(5)**

Solving for K we get

$$K = \frac{\sum_{i=1}^{N} x_i - \sum_{i=1}^{n} x_i}{\sum_{i=1}^{n}(w_i - 1)x_i} \qquad \textbf{(6)}$$

The numerator of *K* is the sum of the cost of energy for sampling units not selected, and the denominator is the sum of the non self representing piece from the sample.

Properites:
Since $n \le N$, then the numerator must be greater than or equal to zero.
Since $w_i \ge 1$ and $x_i \ge 0$ then the denominator must be greater than or equal to zero, which implies that K can never be negative, and the minimum adjusted weight is therefore 1.
For noncertainties the adjusted weight will be greater than or equal to one.
For certainties the adjusted weight equals the original weight of one, i.e. $w_i = 1$ so $1 + K(w_i - 1) = 1$. (Certainties who have weights greater than 1 due to a previous adjustment, like a nonresponse adjustment, will have this weight altered again)